# LOCALIZATION OF ELECTRIC FIELDS, CURRENTS, AND SUPERHEATING INSTABILITY IN A CONDUCTIVE LIQUID ON THE SURFACE OF ELLIPSOIDAL INHOMOGENEITIES

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It is well known [1] that the presence of inhomogeneities in a heated liquid may be the cause of formation of explosive boiling regions. According to [2], such regions may form in the vicinity of filaments heated by electric currents, or in the vicinity of spherical inclusions in the liquid, through which electrical currents also pass [3]. If the liquid surrounding the inhomogeneity is characterized by a nonlinear temperature dependence of electrical conductivity, the cause of explosive heterogeneous liquid boiling may be development of superheating instability, which, according to [4, 5], leads to current pinching on the electrode surfaces. Since macroscopic inclusions in the liquid may be maintained in a stable state by external magnetic [6] or electric [7] fields (the inclusions may serve as a means of discharge initiation in the liquid [8-10]), it is of interest to study in greater detail the localization of fields, currents, and Joulean energy dissipation on the surface of inhomogeneities, as well as the conditions for development of superheating instability on inclusions. It should be noted that these questions were partially considered in [3, 11-14], from which the following conclusions follow: 1) Highly conductive inhomogeneities are more able to concentrate current and Joulean heat liberation than weakly conductive ones; 2) in the vicinity of highly conductive inclusions the current maximum is localized near points (poles) on the liquid-inclusion boundary, while for weakly conductive inclusions the maximum is in the zone of the equatorial line (Fig. 1); an increase in the prolateness of conductive inclusions along the external electric field leads to an increase in concentration of field, current, and Joulean heat dissipation in the surrounding liquid.

If, for example, the external homogeneous electric field $E_1(\infty)$ is directed along the z axis and the electrical conductivity of the inclusions is much greater than that of the liquid ($\sigma_2 >> \sigma_1$), then according to [11-14], the reduced electric field intensity $E_{1z}(x = y = 0, z = \pm c)/E_{1z}(\infty)$ and the reduced current density $j_{1z}(x = y = 0, z = \pm c)/j_{1z}(\infty)$ at points in the liquid ($x = y = 0, z = \pm c$) is inversely proportional to the depolarization coefficient $n^{(z)}$ of the inclusion along the z axis (curve 1, Fig. 2), while the reduced Joulean energy dissipation density $q_V(x = y = 0, z = \pm c)/q_{V1}(\infty)$ is inversely proportional to the parameter $n^{(z)2}$ (curve 2 of Fig. 2). The functions presented are for inhomogeneities of spheroidal forms ($a = b$).

With decrease in the relative inclusion conductivity $\sigma_2/\sigma_1$, as can be seen from the expression for relative electric field intensity at the points ($x = y = 0, z = \pm c$),

$$E_{1z}(x = y = 0, z = \pm c)/E_{1z}(\infty) = \frac{1}{n^{(z)} + \frac{\sigma_1}{\sigma_2}(1 - n^{(z)})}, \tag{1}$$

there is a decrease in the rate of increase of the relative values of current density and energy dissipation density with increase in $c/a$ (in Fig. 2, curves 1', 2' for $\sigma_2/\sigma_1 = 5$, curves 1", 2" for $\sigma_2/\sigma_1 = 1$).

Increased values of volume heat liberation density, which on the one hand, according to [4, 5], are of principal significance in initiation of explosive liquid boiling, and on the other, are inhomogeneously distributed over the surface of the inhomogeneities [11, 14], pose a problem in analyzing the conditions for development of liquid superheating instability in the vicinity of inhomogeneities.

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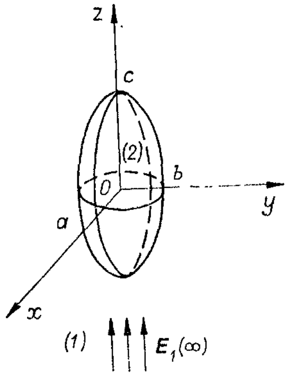


Fig. 1

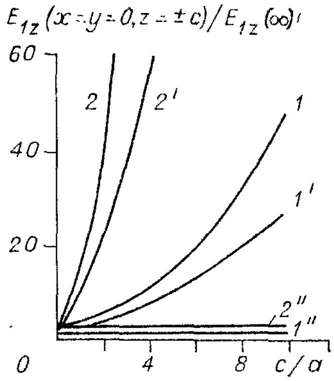


Fig. 2

To determine the increase in superheating instability we will commence from a system of equations describing heat transfer and current in the liquid:

$$\rho c_V \frac{\partial T}{\partial t} - \lambda \Delta T = \mathbf{j}\cdot\mathbf{E} - Q_n(T); \tag{2}$$

$$\nabla\cdot\mathbf{j} = 0,\ \nabla\times\mathbf{E} = 0,\ \mathbf{j} = \sigma(T)\mathbf{E}, \tag{3}$$

where $\rho$, $c_V$, $\sigma(T)$, $\lambda$ are the density, specific heat, electrical conductivity, and thermal conductivity of the liquid; T is liquid temperature; $Q_n(T)$ is the volume heat loss in the liquid boundary layer due to heat exchange with the inclusion material, which can be introduced into Eq. (2) as an exchange term by the method described in [4].

We will assume that there exists some equilibrium state of the medium, which is defined by the system of steady state equations

$$-\lambda_0 \Delta T_0 = \mathbf{j}_0\cdot\mathbf{E}_0 - Q_n(T_0); \tag{4}$$

$$\nabla\cdot\mathbf{j}_0 = 0,\ \nabla\times\mathbf{E}_0 = 0,\ \mathbf{j}_0 = \sigma(T_0)\mathbf{E}_0, \tag{5}$$

and represents the temperature $\delta T$ and field $\delta E$ perturbations in the form

$$\delta T = \delta T_a \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t),\ \delta T = T - T_0,\ i^2 = -1; \tag{6}$$

$$\delta\mathbf{E} = \delta\mathbf{E}_a \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t),\ \delta\mathbf{E} = \mathbf{E} - \mathbf{E}_0, \tag{7}$$

where $\delta T_a$, $\delta E_a$ are the coordinate dependent amplitudes of the perturbations; $\omega$ is the frequency of the perturbations; k is the wave vector; and r is the radius vector.

Substituting Eqs. (6), (7) in Eqs. (2), (3), we obtain the following system relating the temperature and field perturbations:

$$\rho c_V \left[\frac{\delta T}{\delta T_a}\frac{\partial T_a}{\partial t} - i\omega\delta T\right] - \lambda\nabla\cdot\left[\frac{\delta T}{\delta T_a}\nabla(\delta T_a) + i\mathbf{k}\delta T\right] = \tag{8}$$

$$= 2\sigma|_{T=T_0}\cdot\mathbf{E}_0\cdot\delta\mathbf{E} + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\mathbf{E}_0^2\delta T - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0}\delta T,$$

$$\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\left[\frac{\nabla(\delta T_a)}{\delta T_a}\delta T + i\mathbf{k}\delta T\right]\cdot\mathbf{E}_0 + \sigma|_{T=T_0}[\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)\nabla\cdot(\delta\mathbf{E}_a) + \delta\mathbf{E}\cdot i\mathbf{k}] = 0,$$

$$\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)\nabla\times(\delta\mathbf{E}_a) + i\mathbf{k}\times\delta\mathbf{E}.$$

System (8) is extremely difficult to analyze, since it contains terms proportional to the derivatives of the fluctuation amplitudes with respect to coördinates and time. If we assume weak dependence of field and temperature fluctuation amplitudes on coordinates and time, which corresponds mathematically to satisfaction of the inequalities

$$\frac{1}{\delta T_a}\frac{\partial T_a}{\partial t} \ll \omega,\ \frac{|\nabla(\delta T_a)|}{\delta T_a} \ll |\mathbf{k}|,\ \frac{|\nabla\cdot(\delta\mathbf{E}_a)|}{|\delta\mathbf{E}|} \ll |\mathbf{k}|, \tag{9}$$

and physically places limits on the frequency and period of the fluctuations, then system (8) can be represented in the simpler form

$$i\omega\rho c_V - \lambda\mathbf{k}^2 + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\mathbf{E}_0^2 - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0} + 2\sigma|_{T=T_0}\mathbf{E}_0\cdot\delta\mathbf{E}/\delta T = 0, \tag{10}$$

$$\sigma|_{T=T_0}\delta\mathbf{E}\cdot\mathbf{k} + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\delta T\mathbf{E}_0\cdot\mathbf{k} = 0,\quad i\mathbf{k}\times\delta\mathbf{E} = 0.$$

Expressing the field perturbations $\delta E_x$, $\delta E_y$ in Eq. (10) in terms of $\delta E_z$

$$\delta E_x = \frac{k_x}{k_z}\,\delta E_z,\quad \delta E_y = \frac{k_y}{k_z}\,\delta E_z,$$

and expressing δT from the second equation of system (10) in terms of δEz,

$$\delta T = \frac{\sigma\,|_{T=T_0}\,\delta E_z\,\frac{\mathbf{k}^2}{k_z}}{\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}(k_x E_{0x} + k_y E_{0y} + k_z E_{0z})}$$

and substituting in the first equation of system (10), we arrive at the dispersion equation

$$i\omega\rho c_V - \lambda\mathbf{k}^2 + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\mathbf{E}_0^2 - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0} - 2\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\frac{(\mathbf{E}_0\cdot\mathbf{k})^2}{\mathbf{k}^2} = 0. \tag{11}$$

Since with the external field oriented along the z axis the x- and y-components of the field in the liquid in the vicinity of the points (x = y = 0, z = ±c) are equal to zero, Eq. (11) simplifies to the form

$$i\omega\rho c_V - \lambda\mathbf{k}^2 + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c) - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0} - 2\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}\frac{E_{1z}^2(x=y=0,\, z=\pm c)\,k_z^2}{\mathbf{k}^2} = 0. \tag{12}$$

We will consider perturbations perpendicular to the field $\mathbf{E}_1(\infty) = \mathbf{E}_{1z}(\infty)\mathbf{k}$, corresponding to pinching of temperature and current in the medium along the field, where $k_z = 0$, $k_x = k_y = 1$, and Eq. (12) takes on the form

$$i\omega\rho c_V - 2\lambda k_\perp^2 + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c) - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0} = 0. \tag{13}$$

It can be seen from Eq. (13) that upon satisfaction of the inequality

$$\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c) > 2\lambda k_\perp^2 - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0} \tag{14}$$

we have instability with an increment

$$\varepsilon(k_\perp) = \frac{\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c) - 2\lambda k_\perp^2 - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0}}{\rho c_V}. \tag{15}$$

The threshold value of the wave vector $k_\perp^*$, which is defined by the expression

$$k_\perp^* = \sqrt{\frac{1}{2\lambda}\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c) - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0}}, \tag{16}$$

shifts higher with increase in $c/a$ and $\sigma_2/\sigma_1$.

In the instability region the increment $\varepsilon(k_\perp)$ reaches a maximum

$$\varepsilon(k_\perp)_{\max} = \frac{\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c) - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0}}{\rho c_V} \tag{17}$$

at the point $k_\perp = 0$.

For perturbations corresponding to layering in the vicinity of the points (x = y = 0, z = ±c), where $k_x = k_y = 0$, $k_z \neq 0$, the dispersion equation takes the form

$$i\omega\rho c_V - \lambda k_z^2 - \left.\frac{\partial Q_n}{\partial T}\right|_{T=T_0} - \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c), \tag{18}$$

whence it follows that the temperature and current distributions in the medium are stable relative to layering along the z axis.

The dispersion equation for fluctuations related to pinching of current and temperature at points (x = y = 0, z = ±c) near inhomogeneities with electrical conductivity relatively low as compared to the liquid has approximately the same structure as Eq. (18):

$$i\omega\rho c_V - 2\lambda k_{\perp}^2 + \left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0} E_{1z}^2(x=y=0,\, z=\pm c), \tag{19}$$

where

$$E_{1z}(x=y=0,\; z=\pm c) \to 0 \quad \text{for} \quad \sigma_2/\sigma_1 \ll 1.$$

From Eq. (19) we may conclude that in the case of weakly conductive inclusions the most probable consequence of heating of the medium in the vicinity of points ($x = y = 0$, $z = \pm c$) will be stabilization of perturbations, while for the case of highly conductive inhomogeneities, pinching of field, current, and temperature in the direction of the applied external field is most probable.

If we consider further the vicinity of points ($z = 0$) on the surface of ellipsoidal inhomogeneities, then in an external field directed along the z axis, the field intensity at these points, according to [11, 12], will be defined by the expression

$$E_{1z}(z=0) = \frac{E_{1z}(\infty)\,\sigma_1/\sigma_2}{n^{(z)} + (1-n^{(z)})\,\sigma_1/\sigma_2}. \tag{20}$$

If we analyze the limiting cases obtained from general dispersion equation (11) with respect to the parameter $\sigma_2/\sigma_1$ for perturbations perpendicular and parallel to the external field, it develops that in the vicinity of points ($z = 0$) for highly conductive inhomogeneities the most probable consequence of the nonlinear temperature dependence of conductivity will be stabilization of fluctuations, while for weakly conductive inhomogeneities formation of current pinches along the z-axis is most probable.

Comparative analysis of the increases in superheating instability for identical particle form shows that the efficiency of current pinch formation on conductive particles at points ($x = y = 0$, $z = \pm c$) is higher than that of formation on dielectric particles at points ($z = 0$). Therefore, to initiate electrical discharge in liquids with an increasing temperature dependence of electrical conductivity (for example, tap or sea water [15]), it is preferable to use high-conductivity particles, prolate along the electric field.

In conclusion, we note that with increase in particle prolateness along the field, i.e., increase in the parameter $c/a$, according to Eq. (16) the threshold value of the wave vector increases. On the other hand, the quantity $1/k_{\perp}^*$ is limited by the characteristic inhomogeneity size. We will assume that for spheroidal inclusions $1/k_{\perp}^* \leqslant 0.1a$. Then with consideration of the dependence of the field $E_{1z}(x = y = 0,\ z = \pm c)$ on $c/a$ in the vicinity of ideally conductive inclusions ($E_{1z}(x=y=0,\ z=\pm c) \approx 3E_{1z}(\infty)(c/a)^{5/4}$ at $1 \lesssim c/a \lesssim 10$) the limitation on particle diameter $a$ takes on the form

$$a \geqslant \frac{10}{3E_{1z}(\infty)\,(c/a)^{5/4}\sqrt{\frac{1}{2\lambda}\left.\frac{\partial\sigma}{\partial T}\right|_{T=T_0}}}. \tag{21}$$

In the case of water, for which the quantity $\lambda \approx 0.6$ W/(m·K), and $\partial\sigma/\partial T|_{T=T_0}$ at $T_0 = 293$°K, according to [15] is equal to $0.02\ \Omega^{-1}\cdot m^{-1}\cdot K^{-1}$, the value of $a$ at $c/a = 1$, $E_{1z}(\infty) = 1$ kV/cm equals $\approx 2.6\cdot 10^{-4}$ m, and according to Eq. (21) has a tendency to decrease with increase in $E_{1z}(\infty)$, $c/a$ or decrease in $\lambda$.

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# RESONANCE RADIATION COOLING OF A DIATOMIC MOLECULE GAS FLUX

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The possibility of cooling a molecular gas of diatomic dipole molecules during resonance radiation absorption in the P-branch of the vibrational-rotational band was first examined in [1]. In this case the diminution in temperature of the medium is due to the appearance of an energy flux from the molecule translational degrees of freedom to the rotational because of R–T processes, and then to the vibrational. The lifetime of the effect under the action of a radiation pulse on a fixed gas is determined by the vibrational–translational (V–T) relaxation time or by the time of the intramodal vibrational–vibrational (V–V) exchange [1, 2].

The action of continuous resonance radiation on a medium consisting of diatomic dipole molecules and moving at a given velocity can, as will be shown below, also result in a change in the translational temperature and other macroscopic flux parameters. Their change will here be observed during the whole time of exposure. This paper is devoted to an investigation of the flow features of a mixture containing a diatomic dipole molecule gas in a resonance radiation field.

Let us consider the motion of an inviscid, non-heat-conducting gas in a constant-section channel. At a certain part of this channel let continuous radiation at the frequency $\nu_I = (E_{V''} + E_{j''} - E_{V'} - E_{j'})/h$ act on the gas, where h is Planck's constant, while $E_{V''}$, $E_{j''}$ and $E_{V'}$, $E_{j'}$ are the vibrational V and rotational j energies of the upper and lower levels of the absorbing transition [(V', j') → (V'', j'')], respectively.

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